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Properties of Rational Exponents

Property

1. $a^m \cdot a^n = a^{m+n}$ $\quad 12^{\frac{1}{8}} \cdot 12^{\frac{5}{6}} = 12^{\frac{1}{8}+\frac{5}{6}} = 12^{\frac{23}{24}}$

2. $(a^m)^n = a^{mn}$ $\quad (5^{\frac{1}{3}})^2 = 5^{\frac{2}{3}}$

3. $(ab)^m = a^m b^m$ $\quad (5^{\frac{1}{3}} \cdot 7^{\frac{1}{4}})^3 = 5^1 \cdot 7^{\frac{3}{4}}$

4. $a^{-m} = \frac{1}{a^m}, a \neq 0$ $\quad 5^{-\frac{1}{2}} = \frac{1}{5^{\frac{1}{2}}}$

5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ $\quad \frac{10^1}{10^{\frac{2}{5}}} = 10^{1-\frac{2}{5}} = 10^{\frac{3}{5}}$

6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$ $\quad \left(\frac{56^{\frac{1}{4}}}{7^{\frac{1}{4}}}\right)^5 = \left(\frac{\sqrt[4]{56}}{\sqrt[4]{7}}\right)^5 = \left(\sqrt[4]{8}\right)^5 = 8^{\frac{5}{4}}$

PROPERTIES OF RADICALS

Product Property of Radicals	Quotient Property of Radicals
$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$

1. $(6^6 \cdot 5^6)^{-1/6} = (30^6)^{-\frac{1}{6}} = 30^{-1} = \frac{1}{30}$

2. $\frac{\sqrt{245}}{\sqrt{5}} = \sqrt{49} = \pm 7$

Simplifying Rational Exponent Expressions 83

SIMPLEST FORM A radical with index n is in **simplest form** if the radicand has no perfect nth powers as factors and any denominator has been rationalized.

3. $\sqrt[3]{\frac{5}{9}} = \frac{\sqrt[3]{5}}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{\sqrt[3]{15}}{\sqrt[3]{27}} = \frac{\sqrt[3]{15}}{3}$

LIKE RADICALS Radical expressions with the same index and radicand are **like radicals**. ~~To add or subtract like radicals, use the distributive property.~~

4. $6\sqrt[4]{6} + 2\sqrt[4]{6} = 8\sqrt[4]{6}$

5. $\sqrt[3]{8x^7y^3z^{11}}$

$\sqrt[3]{2 \cdot 2 \cdot 2\, xxx\, xxx\, x\, yyy\, zzz\, zzz\, zzz\, zz}$

$2 \cdot x \cdot x\, y\, z\, z\, z\, \sqrt[3]{x \cdot z^2} = 2x^2yz^3\sqrt[3]{xz^2}$

6. $7\sqrt[3]{2a^5} - a\sqrt[3]{128a^2}$

7. $\sqrt[4]{\frac{a^2}{b^6}}$

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